An extension of RB-ATL

Nguyen, Hoang Nga
(presented by Natasha Alechina)

University of Nottingham, UK

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Introduction

- We extend RB-ATL [Alechina et. at. 2010] to express properties of coalitional abilities under flexible resource constraints such as
  - a certain amount of resources is necessary for a coalition of agents *A* to achieve a property ϕ.
  - *A* has a strategy to achieve a property ϕ by an arbitrary amount of resources
  - *A* has a strategy to maintain a property ϕ if *A* has a certain/arbitrary amount of resources

RB-ATL

Syntax

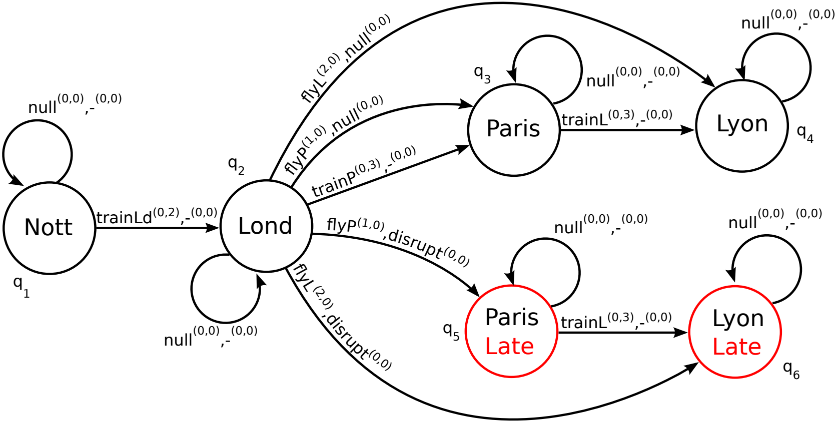
$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\langle A^b \rangle\rangle \bigcirc \varphi \mid \langle\langle A^b \rangle\rangle \Box \varphi \mid \langle\langle A^b \rangle\rangle \varphi \mathcal{U} \psi$$

- A is a non-empty subset of agents
- $b = (b_1, \ldots, b_{|r|})$ is a resource bound where r is the finite set of resources and $b_i \in \mathbb{N}$ for all $i \leq |r|$.

Semantics

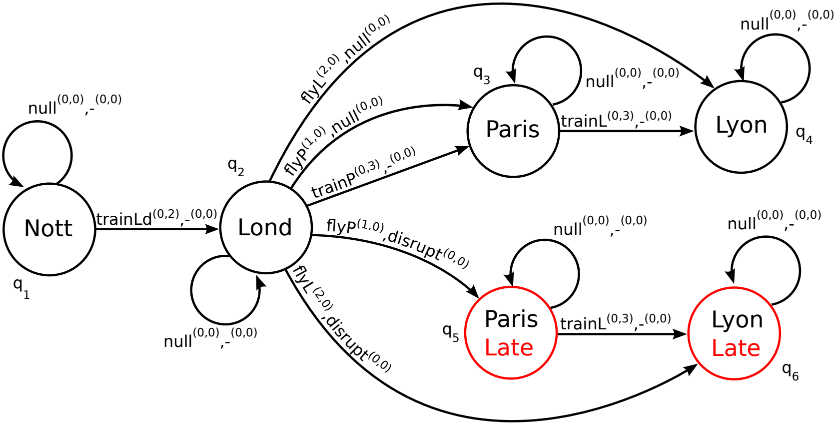
- Formulas are interpreted in RB concurrent game structures.

Example: RB-CGS



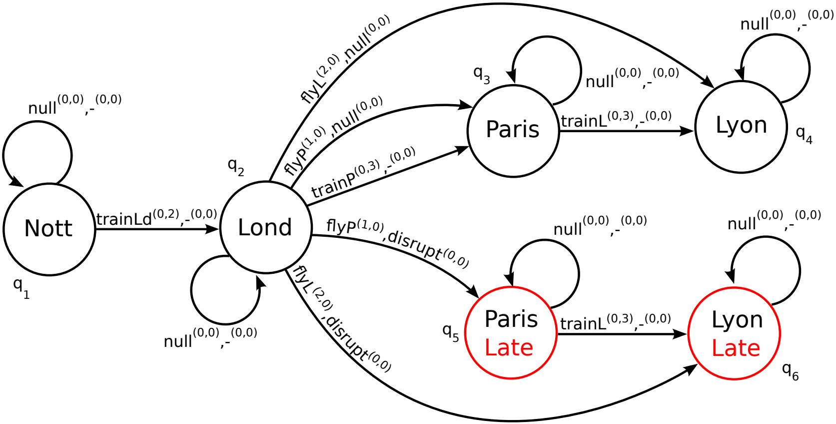
where $-^{(0,0)}$ is either null$^{(0,0)}$ or disrupt$^{(0,0)}$,
$a^{(x,y)}$: x is time on the flight, y on the train.

Example: Properties in RB-ATL (1)



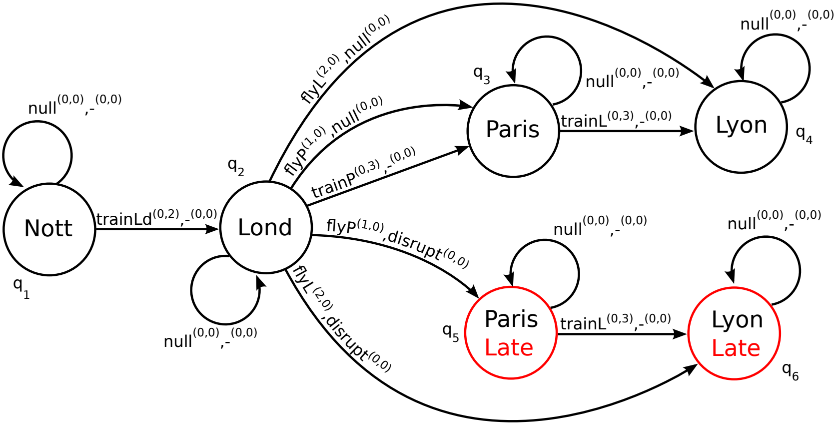
In q_1, can agent 1 get to Lyon on time without spending more than 3 hours on the train and 5 hours on the fight?
$\langle\langle\{1\}^{(5,3)}\rangle\rangle \top \mathcal{U}(\text{Lyon} \wedge \neg\text{Late})$? No

Example: Properties in RB-ATL (2)



In q_1, can agent 1 get to Lyon on time, only by train, without spending more than 10 hours on the train?
$\langle\langle\{1\}^{(0,10)}\rangle\rangle \top \mathcal{U}(\text{Lyon} \wedge \neg\text{Late})$? Yes

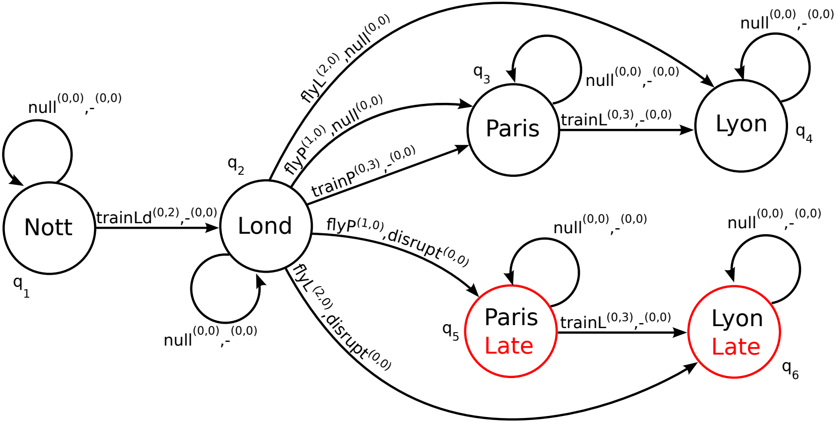
Example: Properties in RB-ATL (3)



In q_1, can agent 1 get to Lyon on time, only by airplane, without spending more than 10 hours on the flight?

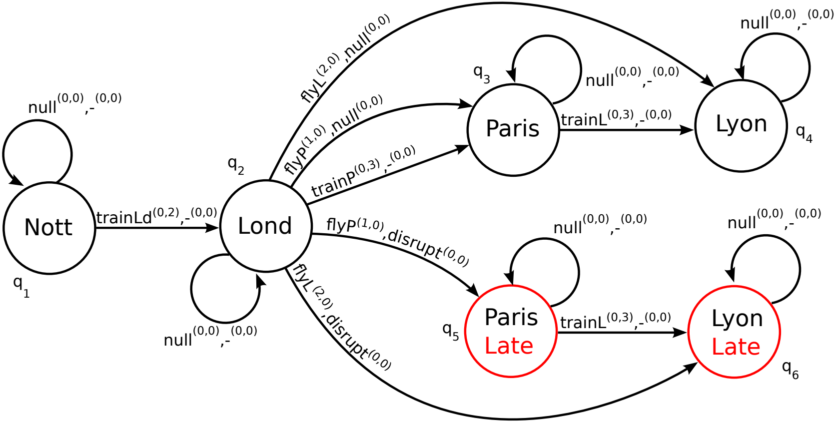
$\langle\langle\{1\}^{(10,0)}\rangle\rangle \top \mathcal{U} (\text{Lyon} \wedge \neg \text{Late})$? No

Example: Properties not expressible in RB-ATL (1)



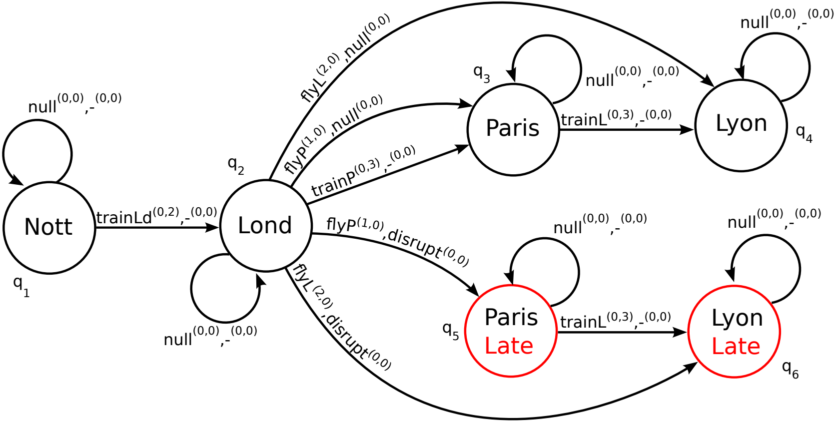
In q_1, can agent 1 get to Lyon (on time)? Yes (No)

Example: Properties not expressible in RB-ATL (2)



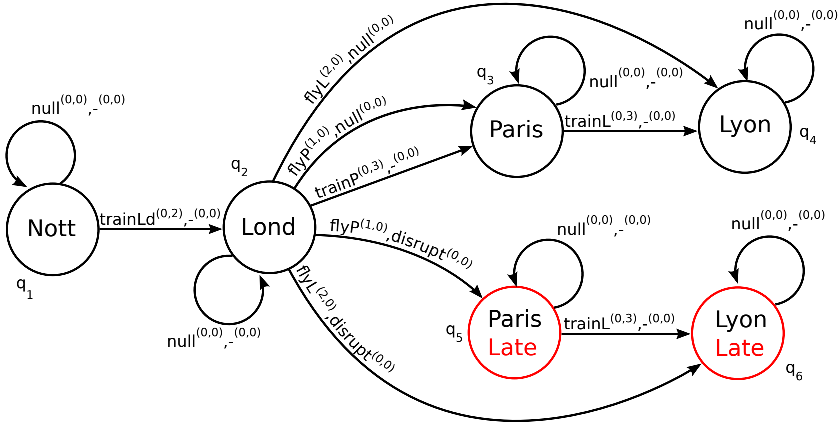
In q_1, can agent 1 get to Lyon only by train (on time)? Yes (Yes)

Example: Properties not expressible in RB-ATL (3)



In q_1, can agent 1 get to Lyon only by airplane (on time)? No (No)

Example: Properties not expressible in RB-ATL (4)



In q_1, can agent 1 get to Lyon (on time) without spending more than 3 hours on the train? Yes (No)

Syntax of RBATL$^{\infty}$

Syntax

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\langle A^b \rangle\rangle \bigcirc \varphi \mid \langle\langle A^b \rangle\rangle \Box \varphi \mid \langle\langle A^b \rangle\rangle \varphi \mathcal{U} \psi$$

- A is a non-empty subset of agents
- $b = (b_1, \ldots, b_{|r|})$ is a resource bound where r is the finite set of resources and $b_i \in \mathbb{N} \cup \{\infty\}$ for all $i \leq |r|$:

$$n \leq \infty \text{ for all } n \in \mathbb{N}$$

$$n + \infty = \infty + n = \infty$$

Example

- $\langle\langle \{1\}^{(\infty,0)} \rangle\rangle \bigcirc$ Lond: agent 1 can get to London by airplain (spend any time on the flight).
- $\langle\langle \{1\}^{(\infty,\infty)} \rangle\rangle \bigcirc$ Lond: agent 1 can get to London.

RB-CGS

A RB-CGS is a tuple $S = (n, r, Q, \Pi, \pi, d, c, \delta)$ where:

- n is the number of agents, $N = \{1, \ldots, n\}$
- r is a non-empty set of resources
- Q is a non-empty set of states
- Π is a finite set of propositional variables
- $\pi : Q \to \wp(\Pi)$ assigns a set of propositional variables to each state
- $d : Q \times N \to \mathbb{N} \setminus \{0\}$ indicates the number of available actions for each agent at a state
- $c : Q \times N \times \mathbb{N} \to \mathbb{B}$ specifies the cost of each action, where $\mathbb{B} = \mathbb{N}^{|r|}$ (costs can't be ∞); there is always $j \leq d(q, i)$ such that $c(q, i, j) \leq 0^{|r|}$
- $\delta : Q \times \mathbb{N}^{|N|} \to Q$ specifies the transition relation

Joint actions and their costs

- A joint action of a coalition A at a state q is

$$\sigma_A = (\sigma_i)_{i \in A} \text{ such that } 1 \leq \sigma_i \leq d(q, i)$$

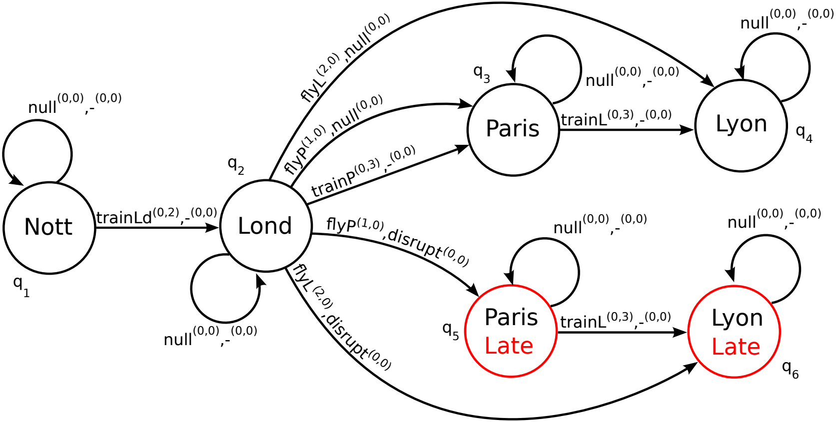
and its cost is

$$cost(q, \sigma_A) = \Sigma_{i \in A} c(q, i, \sigma_i)$$

- $D_A(q)$ denotes the set of all joint actions of A at q
- The set of outcomes of a joint action at a state q is

$$out(q, \sigma_A) = \{\delta(q, (\sigma_A, \sigma_{\bar{A}})) \mid \forall \sigma_{\bar{A}} \in D_{\bar{A}}(q)\}$$

Example: costs and outcomes of joint actions



$cost(q_2, (\text{flyL})) = (2, 0)$ $out(q_2, (\text{flyL})) = \{q_4, q_6\}$

$cost(q_2, (\text{flyL}, \text{disrupt})) = (2, 0)$ $out(q_2, (\text{flyL}, \text{disrupt})) = \{q_6\}$

Strategies and their costs

- A strategy F_A of a coalition A is a partial mapping from Q^+ to the set of joint actions of A such that $F_A(\lambda q) \in D_A(q)$ where $\lambda \in Q^*$
- The set of outcomes $out(q, F_A)$ of a strategy F_A at a state q is a computation $\lambda \in Q^\omega$ such that
  1. $\lambda[0] = q$
  2. $\lambda[i+1] \in out(\lambda[i], F_A(\lambda[0, i]))$ for all $i \geq 0$
- A strategy F_A is a b-strategy iff for every $\lambda \in out(q, F_A)$ and $i \geq 0$

$$cost(\lambda[0, i]) = \Sigma_{j=0}^{j=i} cost(\lambda[j], F_A(\lambda[0, j])) \leq b$$

Example: ‘train’ strategy

A strategy for agent 1 to get to Lyon on time:

λ	$F_{\{1\}}(\lambda)$
q_1	trainLd
$q_1 q_2$	trainP
$q_1 q_2 q_3$	trainL
$q_1 q_2 q_3 q_4 q_4^*$	null

- $out(q_1, F_{\{1\}}) = \{q_1 q_2 q_3 q_4 q_4 \ldots\}$
- $cost(q_1 q_2 q_3 q_4 q_4^*) = (0, 8)$
- $F_{\{1\}}$ is a $(0, 8)$-strategy

Example: 'almost-flying' strategy

Another strategy for agent 1:

λ	$F'_{\{1\}}(\lambda)$
q_1	trainLd
$q_1 q_2$	flyL
$q_1 q_2 q_4 q_4^*$	*null*
$q_1 q_2 q_6 q_6^*$	*null*

- $out(q, F'_{\{1\}}) = \{q_1 q_2 q_4 q_4, \ldots, \ q_1 q_2 q_6 q_6 \ldots\}$
- $cost(q_1 q_2 q_4 q_4^*) = cost(q_1 q_2 q_6 q_6^*) = (2, 2)$
- $F'_{\{1\}}$ is a $(2, 2)$-strategy

Example: joint strategy

Joint strategy for 1 and 2:

λ	$F_{\{1,2\}}(\lambda)$
q_1	$(trainLd, null)$
$q_1 q_2$	$(flyL, null)$
$q_1 q_2 q_4 q_4^*$	$(null, null)$

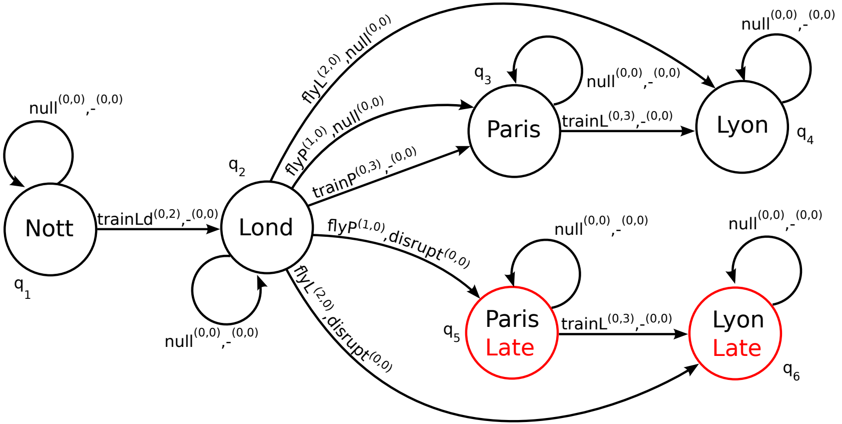
- $out(q, F_{\{1,2\}}) = \{q_1 q_2 q_4 q_4 \ldots\}$
- $cost(q_1 q_2 q_4 q_4^*) = (2, 4)$
- $F_{\{1,2\}}$ is a $(2, 2)$-strategy

Semantics of RB-ATL$^\infty$

Given a RB-CGS $S = (n, r, Q, \Pi, \pi, d, c, \delta)$

- $S, q \models p$ iff $p \in \pi(q)$
- $S, q \models \neg\varphi$ iff $S, q \not\models \varphi$
- $S, q \models \varphi \vee \psi$ iff $S, q \models \varphi$ or $S, q \models \psi$
- $S, q \models \langle\langle A^b \rangle\rangle \bigcirc \varphi$ iff $\exists \sigma_A \in D_A(q)$ such that $cost(q, \sigma_A) \leq b$ and $\forall q' \in out(q, \sigma_A)$, $S, q' \models \varphi$
- $S, q \models \langle\langle A^b \rangle\rangle \Box \varphi$ iff $\exists$ a b- strategy F_A such that $\forall \lambda \in out(q, F_A)$, $S, \lambda[i] \models \varphi$ for all $i \geq 0$
- $S, q \models \langle\langle A^b \rangle\rangle \varphi \mathcal{U} \psi$ iff $\exists$ a b-strategy F_A such that $\forall \lambda \in out(q, F_A)$, $\exists i \geq 0$: $S, \lambda[i] \models \psi$ and $S, \lambda[j] \models \varphi$ for all $j \in \{0, \ldots, i-1\}$

Example: Properties not expressible in RB-ATL (1)

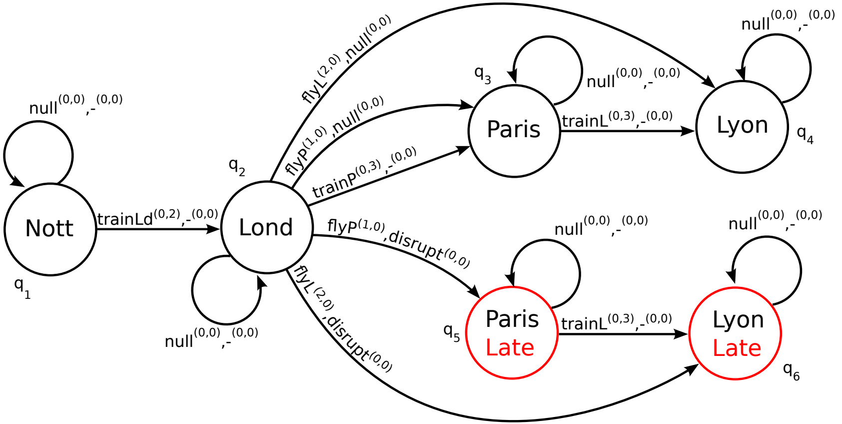


In q_1, can agent 1 get to Lyon (on time)?

$S, q_1 \models \langle\langle \{1\}^{(\infty,\infty)} \rangle\rangle \top \mathcal{U} \text{Lyon}$

$(S, q_1 \models \langle\langle \{1\}^{(\infty,\infty)} \rangle\rangle \top \mathcal{U} \text{Lyon} \wedge \neg \text{Late})$

Example: Properties not expressible in RB-ATL (2)

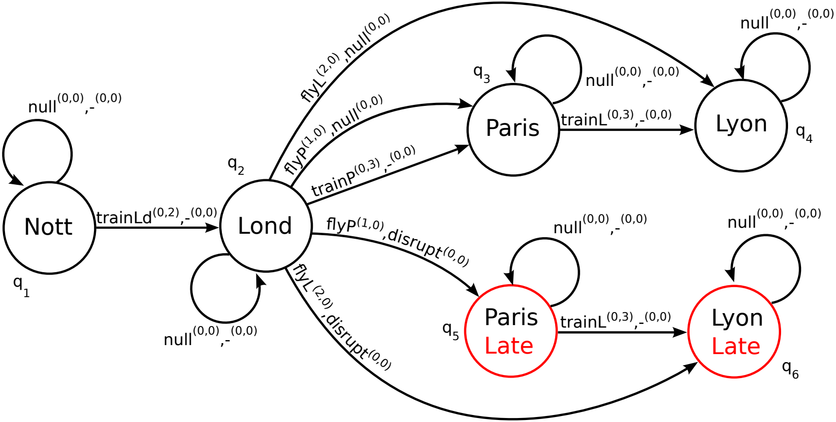


In q_1, can agent 1 get to Lyon only by train (on time)? Yes (Yes)

$S, q_1 \models \langle\langle\{1\}^{(0,\infty)}\rangle\rangle \top \mathcal{U} \text{Lyon}$

$(S, q_1 \models \langle\langle\{1\}^{(0,\infty)}\rangle\rangle \top \mathcal{U} \text{Lyon} \wedge \neg \text{Late})$

Example: Properties not expressible in RB-ATL (3)

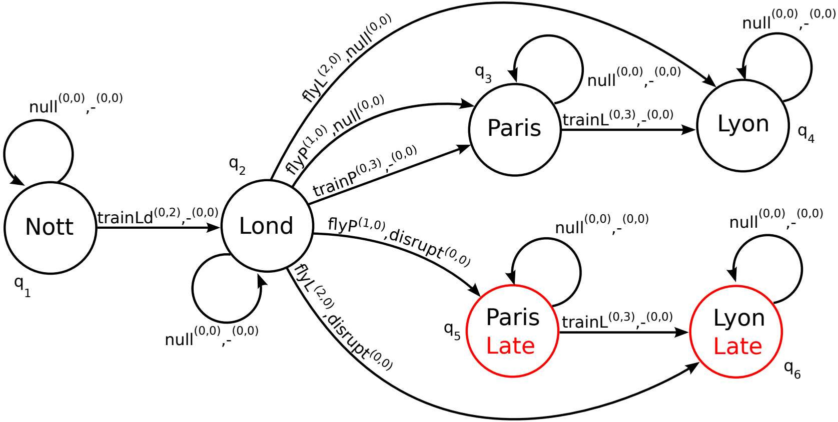


In q_1, can agent 1 get to Lyon only by airplane (on time)? No (No)

$S, q_1 \not\models \langle\langle\{1\}^{(\infty,0)}\rangle\rangle \top \mathcal{U} \text{Lyon}$

$(S, q_1 \not\models \langle\langle\{1\}^{(\infty,0)}\rangle\rangle \top \mathcal{U} \text{Lyon} \wedge \neg \text{Late})$

Example: Properties not expressible in RB-ATL (4)



In q_1, can agent 1 get to Lyon (on time) without spending more than 3 hours on the train? Yes (No)

$S, q_1 \models \langle\langle \{1\}^{(\infty,3)} \rangle\rangle \top \mathcal{U} \text{Lyon}$

$(S, q_1 \not\models \langle\langle \{1\}^{(\infty,3)} \rangle\rangle \top \mathcal{U} \text{Lyon} \wedge \neg \text{Late})$

Some results of RB-ATL$^\infty$

Theorem

The deductive system for RB-ATL$^\infty$ is sound and complete.

- Differences from the proof of RB-ATL: during the construction of models for consistent formulas, it is sometimes required to guess costs of actions by taking into account bounds in negated formulas starting with $\langle\langle A^b \rangle\rangle$.

Theorem

RB-ATL$^\infty$ is decidable.

Future work

- Considering the model-checking problem for RB-ATL$^{\infty}$
- Extending RB-ATL$^{\infty}$ for the case where actions may both consume and produce resources

Axiomatisation for RB-ATL$^\infty$ (1)

(PL) Tautologies of Propositional Logic

($\bot$) $\neg\langle\langle A^b \rangle\rangle \bigcirc \bot$

($\top$) $\langle\langle A^b \rangle\rangle \bigcirc \top$

(B) $\langle\langle A^b \rangle\rangle \bigcirc \varphi \rightarrow \langle\langle A^d \rangle\rangle \bigcirc \varphi$
where $b \leq d$

(S) $\langle\langle A_1^{b_1} \rangle\rangle \bigcirc \varphi \wedge \langle\langle A_2^{b_2} \rangle\rangle \bigcirc \psi \rightarrow \langle\langle (A_1 \cup A_2)^{b_1+b_2} \rangle\rangle \bigcirc (\varphi \wedge \psi)$
where $A_1 \cap A_2 = \emptyset$

($S_\emptyset$) $\langle\langle \emptyset^{b_1} \rangle\rangle \bigcirc \varphi \wedge \langle\langle \emptyset^{b_2} \rangle\rangle \bigcirc \psi \rightarrow \langle\langle \emptyset^{b_1} \rangle\rangle \bigcirc (\varphi \wedge \psi)$
where $b_1 \leq b_2$

(S_N) $\langle\langle N^{b_1} \rangle\rangle \bigcirc \varphi \wedge \langle\langle \emptyset^{b_2} \rangle\rangle \bigcirc \psi \rightarrow \langle\langle N^{b_1} \rangle\rangle \bigcirc (\varphi \wedge \psi)$
where $b_1 \leq b_2$

($FP_\Box$) $\langle\langle A^b \rangle\rangle \Box \varphi \leftrightarrow \varphi \wedge (\langle\langle A^b \rangle\rangle \bigcirc \Box \varphi \vee \langle\langle A^{\bar{0}_b} \rangle\rangle \bigcirc (\langle\langle A^b \rangle\rangle \Box \varphi))$

Axiomatisation for RB-ATL$^\infty$ (2)

$$\begin{array}{rl}
(\mathrm{FP}_{\mathcal{U}}) & \langle\langle A^b\rangle\rangle\varphi\mathcal{U}\psi \leftrightarrow \\
& \psi \vee (\varphi \wedge (\langle\langle A^b\rangle\rangle \bigcirc \varphi\mathcal{U}\psi \vee \langle\langle A^{\bar{0}_b}\rangle\rangle \bigcirc (\langle\langle A^b\rangle\rangle\varphi\mathcal{U}\psi))) \\
(N_{\bigcirc}) & \langle\langle \emptyset^b\rangle\rangle \bigcirc \varphi \leftrightarrow \neg\langle\langle N^b\rangle\rangle \bigcirc (\neg\varphi) \\
(N_{\Box}) & \langle\langle \emptyset^b\rangle\rangle\Box\varphi \leftrightarrow \varphi \wedge \neg\langle\langle N^b\rangle\rangle\top\mathcal{U}\neg\varphi \\
(N_{\mathcal{U}}) & \langle\langle \emptyset^b\rangle\rangle\varphi\mathcal{U}\psi \leftrightarrow \neg(\langle\langle N^b\rangle\rangle\neg\psi\mathcal{U}\neg(\varphi\vee\psi) \vee \langle\langle N^b\rangle\rangle\Box\neg\psi)
\end{array}$$

where $\langle\langle \emptyset^b\rangle\rangle$ is the quantifier over all strategies of N which spend less than b amount of resources and

$$\begin{array}{l}
\langle\langle A^b\rangle\rangle \bigcirc \Box\varphi = \bigvee_{b_1 +^\infty b_2 = b}\langle\langle A^{b_1}\rangle\rangle \bigcirc \langle\langle A^{b_2}\rangle\rangle\Box\varphi \\
\neg\langle\langle A^b\rangle\rangle \bigcirc \Box\varphi = \bigwedge_{b_1 +^\infty b_2 = b}\neg\langle\langle A^{b_1}\rangle\rangle \bigcirc \langle\langle A^{b_2}\rangle\rangle\Box\varphi \\
\langle\langle A^b\rangle\rangle \bigcirc \varphi\mathcal{U}\psi = \bigvee_{b_1 +^\infty b_2 = b}\langle\langle A^{b_1}\rangle\rangle \bigcirc \langle\langle A^{b_2}\rangle\rangle\varphi\mathcal{U}\psi \\
\neg\langle\langle A^b\rangle\rangle \bigcirc \varphi\mathcal{U}\psi = \bigwedge_{b_1 +^\infty b_2 = b}\neg\langle\langle A^{b_1}\rangle\rangle \bigcirc \langle\langle A^{b_2}\rangle\rangle\varphi\mathcal{U}\psi \\
\langle\langle \emptyset^b\rangle\rangle \bigcirc \Box\varphi = \bigwedge_{b_1 +^\infty b_2 = b}\langle\langle \emptyset^{b_1}\rangle\rangle \bigcirc \langle\langle \emptyset^{b_2}\rangle\rangle\Box\varphi \\
\neg\langle\langle \emptyset^b\rangle\rangle \bigcirc \Box\varphi = \bigvee_{b_1 +^\infty b_2 = b}\langle\langle \emptyset^{b_1}\rangle\rangle \bigcirc \langle\langle \emptyset^{b_2}\rangle\rangle\Box\varphi
\end{array}$$

Axiomatisation for RB-ATL$^\infty$ (3)

(MP) $\dfrac{\varphi, \varphi \to \psi}{\psi}$

($\langle\langle A^b \rangle\rangle \bigcirc$-Monotonicity) $\dfrac{\varphi \to \psi}{\langle\langle A^b \rangle\rangle \bigcirc \varphi \to \langle\langle A^b \rangle\rangle \bigcirc \psi}$

($\langle\langle \emptyset^b \rangle\rangle \Box$-Necessitation) $\dfrac{\varphi}{\langle\langle \emptyset^b \rangle\rangle \Box \varphi}$

($\langle\langle A^b \rangle\rangle \Box$-Induction) $\dfrac{\theta \to (\varphi \wedge (\langle\langle A^b \rangle\rangle \bigcirc \Box \varphi \vee \langle\langle A^{\bar{0}_b} \rangle\rangle \bigcirc \theta))}{\theta \to \langle\langle A^b \rangle\rangle \Box \varphi}$

($\langle\langle A^b \rangle\rangle \mathcal{U}$-Induction)

$$\frac{(\psi \vee (\varphi \wedge (\langle\langle A^b \rangle\rangle \bigcirc \varphi \mathcal{U} \psi) \vee \langle\langle A^{\bar{0}_b} \rangle\rangle \bigcirc \theta))) \to \theta}{\langle\langle A^b \rangle\rangle \varphi \mathcal{U} \psi \to \theta}$$