

Logic meets cognition: Empirical reasoning in games

Sujata Ghosh, Ben Meijering, Rineke Verbrugge

University of Groningen

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Warning!

a preliminary study towards reaching an ambitious goal ...



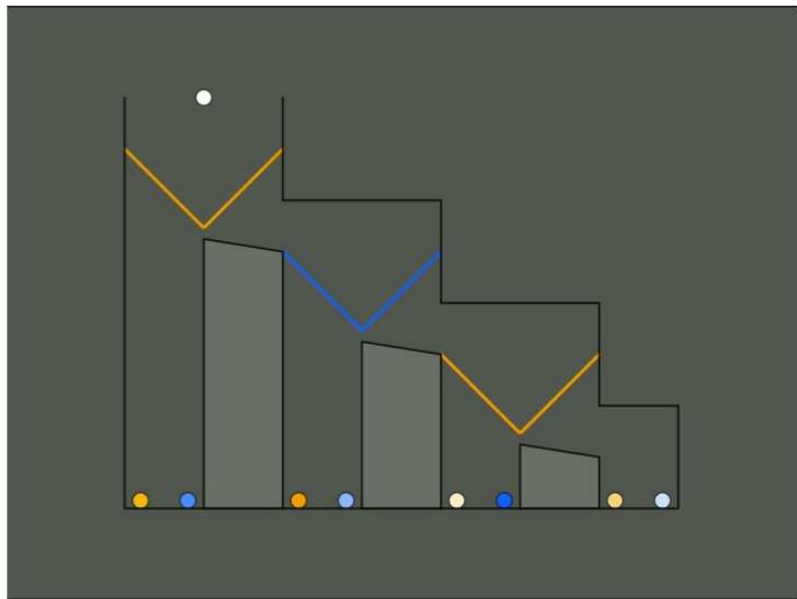
A milieu

- ▶ Experimental studies [Meijering et al., CogSci2010]
- ▶ Cognitive models [van Maanen et al., CogMod2010]
- ▶ Logical framework

Experimental studies

- ▶ Main motivation
 - ▶ higher order social reasoning is not too difficult
 - ▶ embed within a context
 - ▶ scenario: marble drop game
- ▶ Our take
 - ▶ performing strategic reasoning in games

Marble drop game

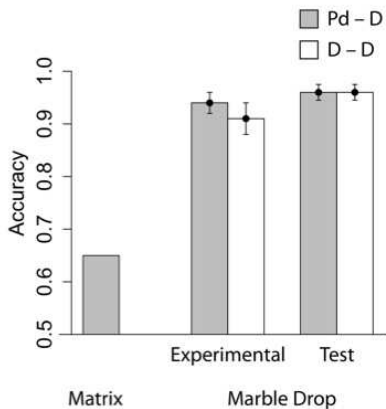


Matrix game

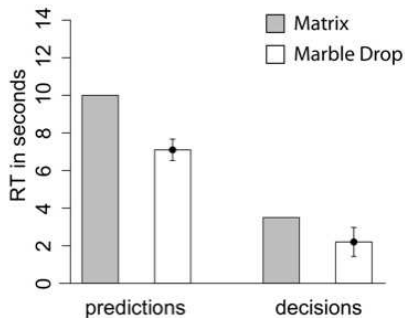
A			D
	3 1		2 2
	↓ Player 1 decides		↑ Player 1 decides
	4 3	→	1 4
B		Player 2 decides	C

(Hedden and Zhang, 2002)

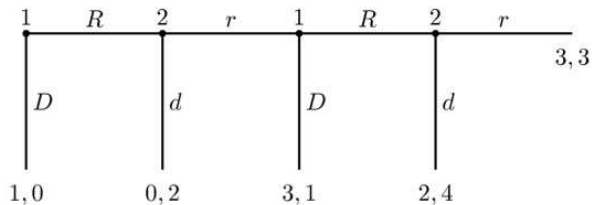
Predictions and decisions



Speed vs. Accuracy



Centipede game



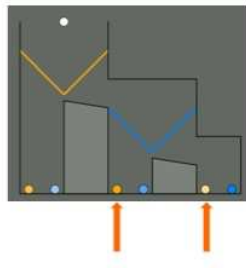
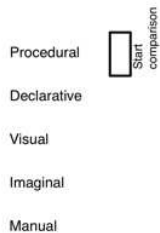
ACT-R model

explaining default behavior following the higher-order social reasoning strategy (backward induction).

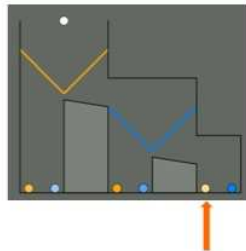
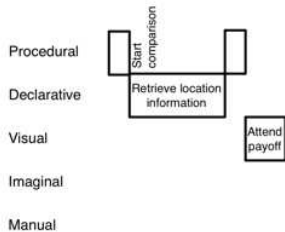
- ▶ model overview
 - ▶ compare two marbles
 - ▶ decide on next comparison
- ▶ requirements
 - ▶ declarative memory
 - ▶ problem state representation

Model structure

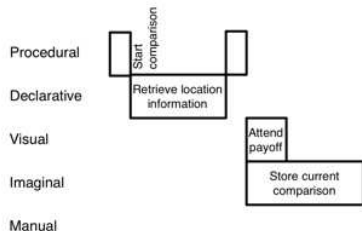
Model structure



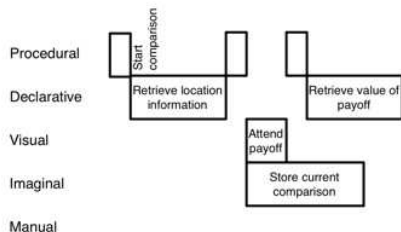
Model structure



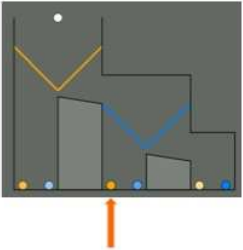
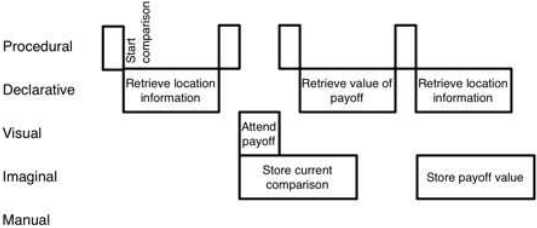
Model structure



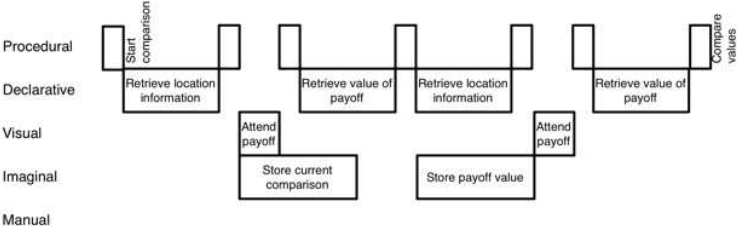
Model structure



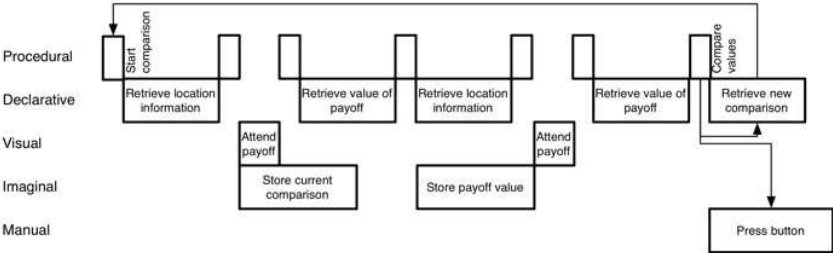
Model structure



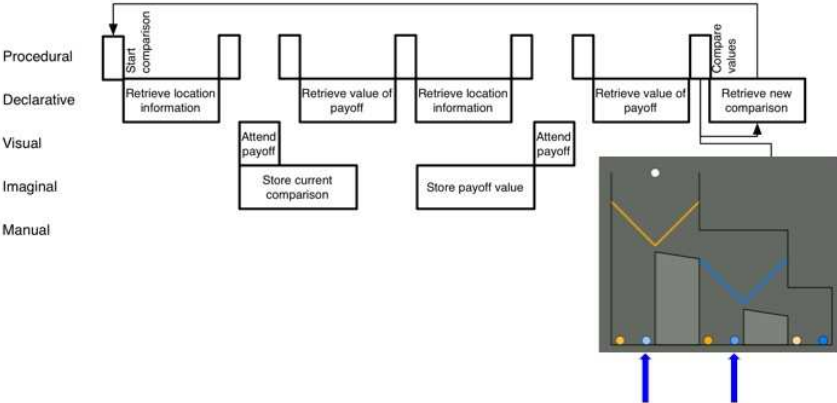
Model structure



Model structure



Model structure



Meeting point

so far so good



Question: What has logic got to do with all these studies?

Meeting point

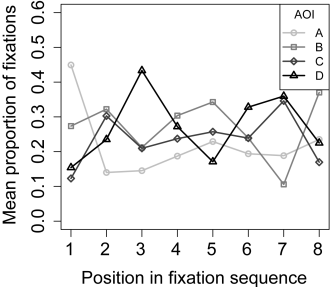
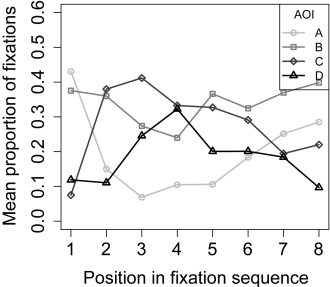
so far so good



Question: What has logic got to do with all these studies?

How do the participants reason? What sort of strategies do they come up with? Do they always follow backward induction strategy?

Eye-tracking studies



Rational agents in real world

Practicalities of players interacting in the real world.

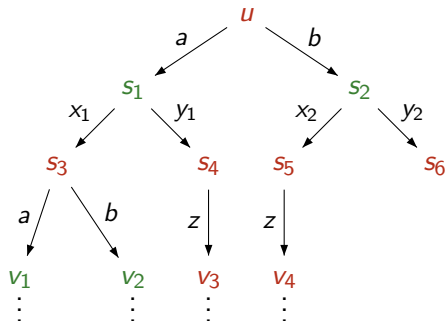
- ▶ Players are agents with limited computation resources.
- ▶ Players employ bounded memory strategies.
- ▶ Strategies are better viewed as relations constraining moves rather than complete functions.
- ▶ **bottom up** vs. **top down**

Extensive form games

Game model - rooted tree where nodes are labelled with players.

P - countable set of observables

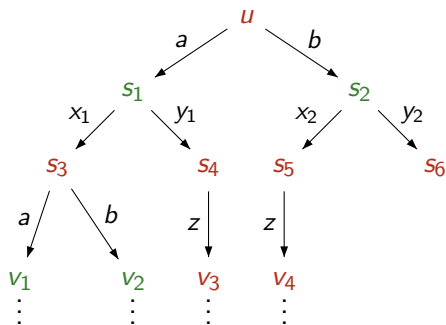
$$V : \text{Nodes} \rightarrow 2^P$$



Extensive form game tree

Extensive form games

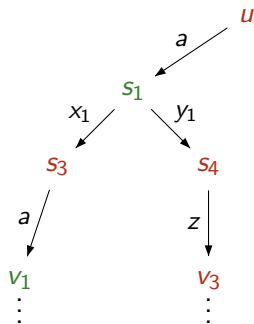
Strategies of players - subtrees of the game tree.



Extensive form game tree

Extensive form games

Strategies of players - subtrees of the game tree.



A strategy of player 1

Strategy specification (Ramanujam and Simon, 2008)

$$\text{Strat}^i(P^i) := [\psi \mapsto a]^i$$

Interpretation

- ▶ $[\psi \mapsto a]^i$: If the observable ψ holds then choose action a (**positional strategies**).

Syntax for ψ

$$p \mid \neg\psi \mid \psi_1 \vee \psi_2 \mid \langle a^+ \rangle\psi \mid \langle a^- \rangle\psi.$$

Strategy specification (Ramanujam and Simon, 2008)

$$\text{Strat}^i(P^i) := [\psi \mapsto a]^i \mid \sigma_1 + \sigma_2 \mid \sigma_1 \cdot \sigma_2$$

Interpretation

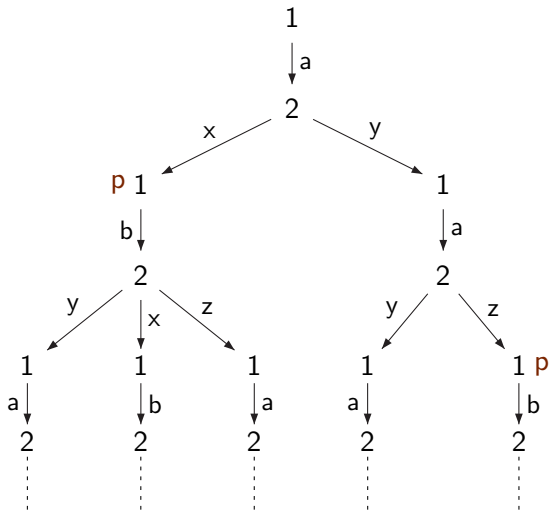
- ▶ $[\psi \mapsto a]^i$: If the observable ψ holds then choose action a (**positional strategies**).
- ▶ $\sigma_1 + \sigma_2$: Disjunction.
- ▶ $\sigma_1 \cdot \sigma_2$: Conjunction.

Strategy specification

- ▶ Strategy specifications need not define complete strategies.
- ▶ Define when a (relational) strategy satisfies a specification.

Strategy conforming to a specification

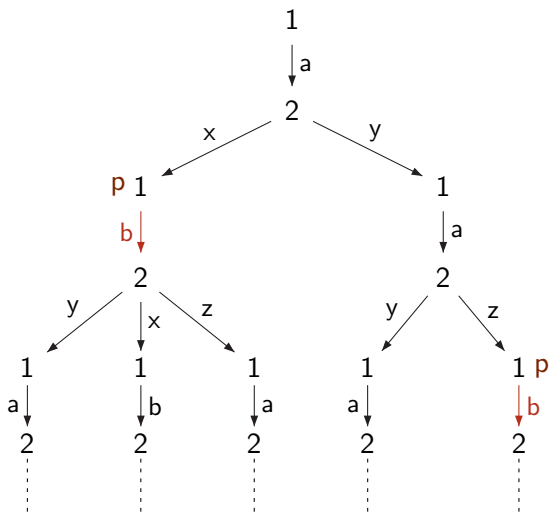
Player 1 strategy.



Strategy conforming to a specification

Player 1 strategy.

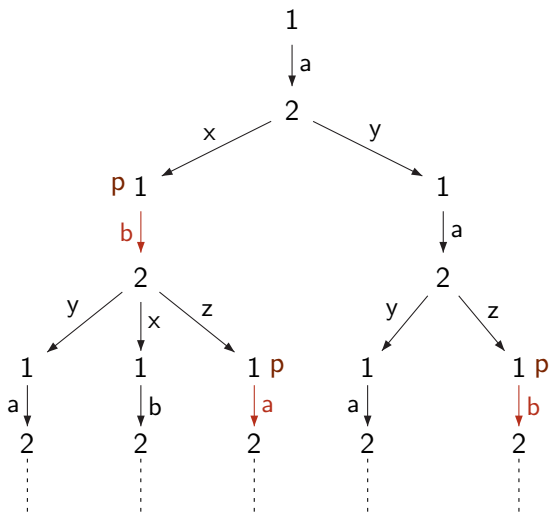
$[p \mapsto b]^1$



Strategy conforming to a specification

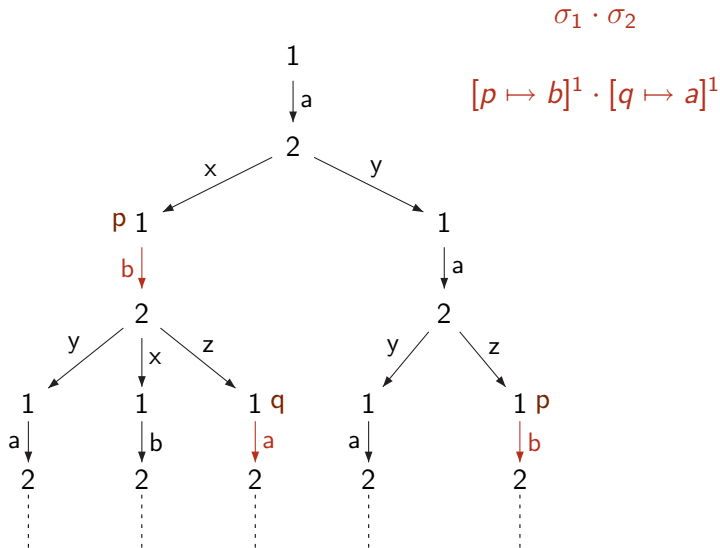
Player 1 strategy.

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Strategy conforming to a specification

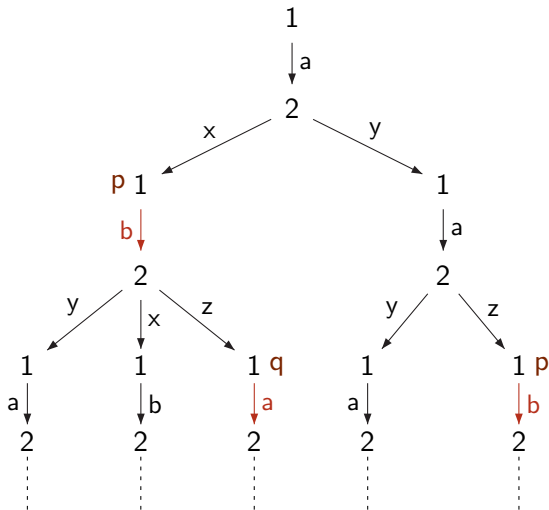
Player 1 strategy.



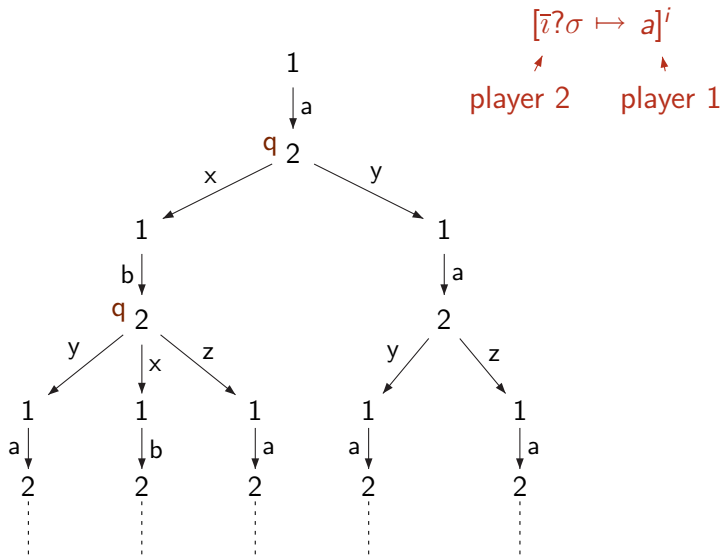
Strategy conforming to a specification

Player 1 strategy.

$\sigma_1 + \sigma_2$

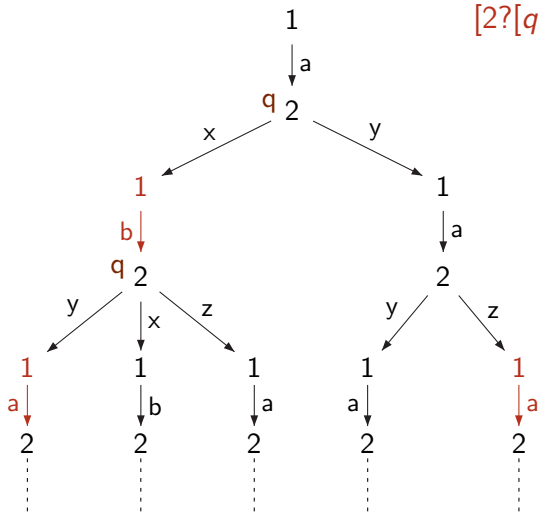


Response strategy

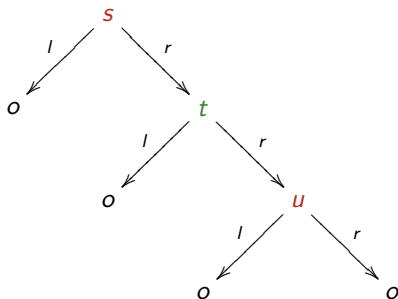


Response strategy

$$[\bar{i}?\sigma \mapsto a]^i$$
$$[2?[q \mapsto x]^2 \mapsto b]^1$$



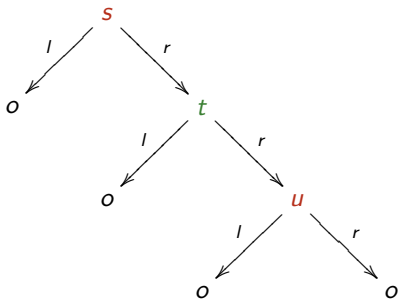
Reasoning in centipede games



- ▶ **forward reasoning:** $\langle [1?[root \wedge \mathbf{turn}_1 \mapsto r]^1 \wedge \langle r^- \rangle root \wedge \mathbf{turn}_2 \mapsto r]^2, [2?[\langle r^- \rangle root \wedge \mathbf{turn}_2 \mapsto r]^2 \wedge \langle r^- \rangle \langle r^- \rangle root \wedge \mathbf{turn}_1 \mapsto l]^1 \rangle$.

if player 1 makes the move r at the root node, player 2 will respond with playing r , and if player 2 behaves like that, player 1 would play l .

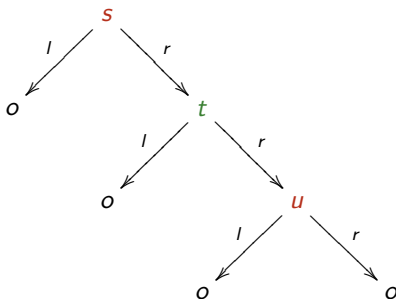
Reasoning in centipede games



- ▶ **backward reasoning:** $\langle [1?[\langle r^- \rangle \langle r^- \rangle \text{root} \wedge \mathbf{turn}_1 \mapsto r]^1 \wedge \mathbf{turn}_2 \mapsto r]^2, [2?[\langle r^- \rangle \text{root} \wedge \mathbf{turn}_2 \mapsto r]^2 \wedge \text{root} \wedge \mathbf{turn}_1 \mapsto l]^1 \rangle$.

if player 1 makes the move r at the u node (if the game reaches there) , player 2 will play r when her turn comes, and if player 2 behaves like that, player 1 would play l at the start node.

Reasoning in centipede games



- ▶ **combined reasoning:** $\langle [root \wedge \mathbf{turn}_1 \mapsto r]^1, [1?[\langle r^- \rangle \langle r^- \rangle root \wedge \mathbf{turn}_1 \mapsto r]^1 \wedge \langle r^- \rangle root \wedge \mathbf{turn}_2 \mapsto r]^2 \rangle$.

player 1 would play r at the root node, and then player 2 will play r after that, since if the game reaches the u node, player 1 will play r .

Future directions

- ▶ To answer the **why** question: How to bring in the notion of pay-off comparison?

Future directions

- ▶ To answer the **why** question: How to bring in the notion of pay-off comparison?
- ▶ How to relate to cognitive modelling?
- ▶ How to relate to the experimental studies?
- ▶ More in-depth study of the logic!